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Design, analysis and testing of a force sensor for use in teaching and research

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ABSTRACT: In this article, the author presents a project, the results of which have been integrated into instruction and research. A force sensor for laboratory experiments in fluid mechanics was required. So, several concepts were explored and tested as a class exercise in order to design, analyse, build and evaluate this device. The most successful design was based upon the Whittmore-Petrenko proving ring. The elastic behaviour of the ring was analysed using Castigliano's second theorem and the results so obtained were found to be within 1% of those from testing. The ring is currently used in laboratory exercises and in research projects in fluid mechanics.

INTRODUCTION

A force sensor was designed, analysed, built and tested to be used in laboratory experiments in order to measure forces applied to solid objects that were immersed in a fluid. This sensor is a variation of the Whittmore-Petrenko proving ring [9][10].

The Whittmore-Petrenko proving ring, now known simply as the proving ring, is a metal ring that is equipped with a means of measuring its deflection under load. The concept and design were created originally by Whittmore and Petrenko at the (US) National Bureau of Standards, which is now called the National Institute of Standards and Technology [9][10]. The proving ring is used to measure force [5].

In its original design, the proving ring consists of two main elements, namely:

- The ring itself;
- The diameter-measuring system.

The ring is made from an elastic material, such as a steel alloy, and has a known diameter [9][10]. The measuring device is located in the center of the ring. A sketch of the original design is shown in Figure 1.

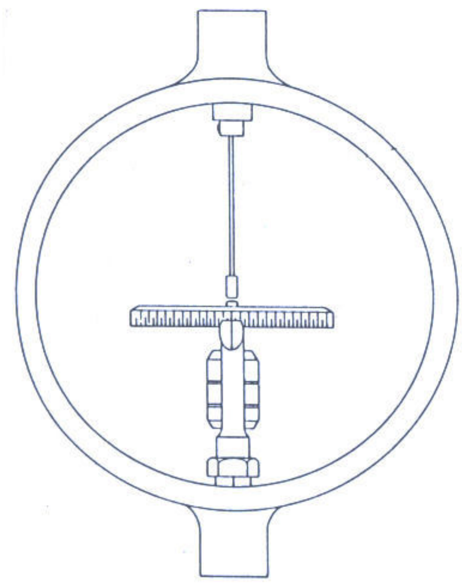

Figure 1: The Whittmore-Petrenko proving ring [1].

As tensile or compressive forces are applied to the ring through the knobs at the top and bottom, known as the external bosses, they cause the diameter of the ring to change: when subjected to vertical tension, the ring will stretch vertically and shrink horizontally; and, when subjected to vertical compression, it will swell horizontally and shrink vertically, as illustrated in Figure 2. Such changes in the diameter are called *deflections of the ring* [9][10]. These deflections are measured using a micrometer screw and a vibrating reed that are mounted within the ring and along its vertical diameter (Figure 1).

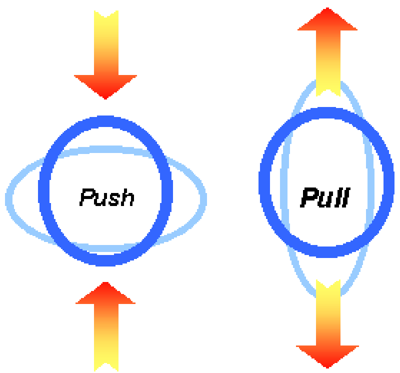

Figure 2: Changes in the shape of the ring [9].

The proving ring that was designed and built in the laboratory at Indiana University-Purdue University Fort Wayne in Fort Wayne, USA, replaces the micrometer and reed in the Whittmore-Petrenko proving ring with a set of electrical resistance strain gauges. A thin ring was used as a circular-shaped load cell. It utilises four strain gauges as secondary transducers, as sketched by Beckwith and Marangoni [1].

Two transducers are mounted on the inner surface of this ring; the other two are mounted on its outer surface; and all four are connected to the Wheatstone bridge of a strain indicator as indicated schematically in Figure 3 [12]. Thus, the ring so constructed becomes a load cell that measures strain instead of deflection. When in use in the laboratory, the force sensor is rigidly supported from above and hangs in a vertical plane as shown in Figure 4.

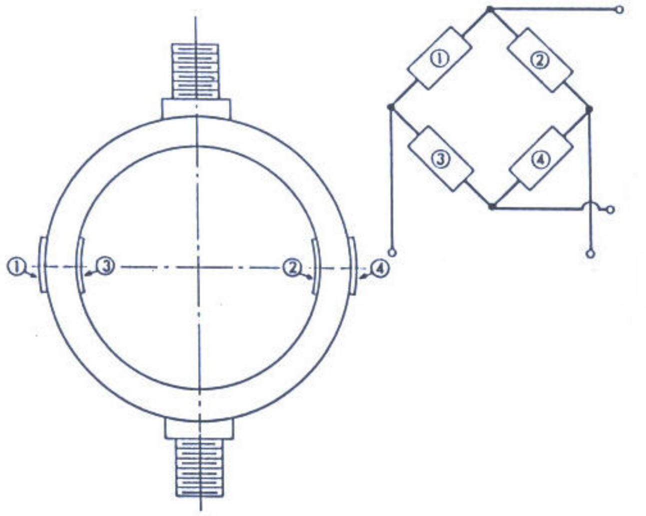

Figure 3: The ring with mounted gauges [1].

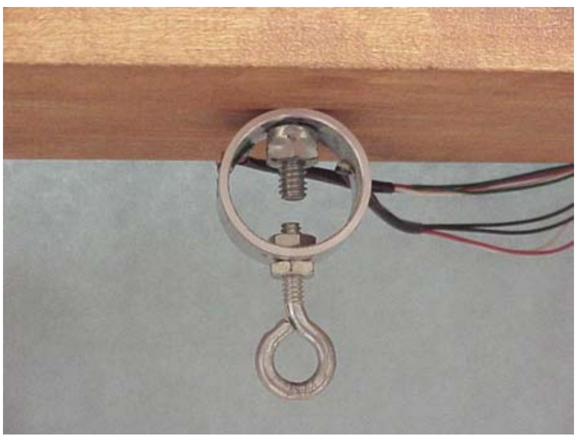

Figure 4: The ring used in the laboratory.

ANALYSIS OF THE RING

In order to analyse the ring to show that its mechanical behaviour is linearly elastic, the conventional model of the loaded ring was used, as shown in Figure 5(a) [7].

After utilising symmetry about the vertical and horizontal axes, the upper-right quadrant of the ring was selected for study, as shown in Figure 5(b) [7]. It can be seen that the problem is statically indeterminate to the first degree.

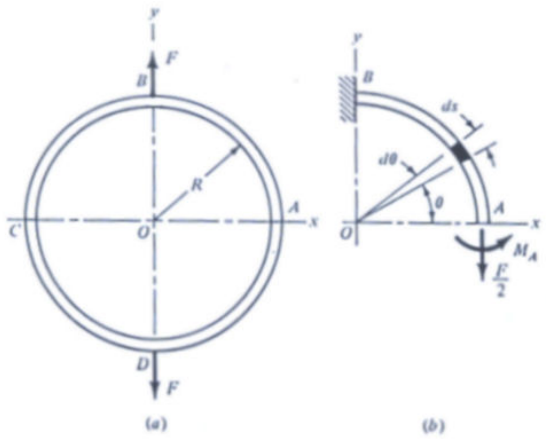

Figure 5: Typical model of the ring used for analysis [7].

Castigliano's second theorem was used to derive an approximation to the deflection of a thin ring that is subjected to an axial force F, as shown in Figure 5 [3][4][6].

Referring to Figure 5(b) [7], an expression for the bending moment, M(θ), at a cross section of the curved beam that is at an angle θ away from the x-axis, can be obtained from the equilibrium of the moments applied to the lower segment of the quadrant about that cross section. Doing so leads to:

$$M(\theta) = M_A - \frac{FR}{2}(1-\cos\theta) \tag{1}$$

The strain energy U is given by:

$$U \equiv \int_0^{\pi/2} \frac{M^2 R d\theta}{2EI} \tag{2}$$

From Castigliano's theorem [3][4][6], the vertical deflection of this quadrant of the ring is given by:

$$\delta \equiv \frac{\partial U}{\partial F} = \int_0^{\pi/2} \frac{MR}{EI}\frac{\partial M}{\partial F} d\theta \tag{3}$$

where, by using Eq. (1), one gets:

$$\frac{\partial M}{\partial F} = \frac{\partial M_A}{\partial F} + \frac{R}{2}(\cos\theta - 1) \tag{4}$$

The first term in Eq. (4) is unknown and must be determined by other means. Noting that the cross-section at point A does not rotate under this loading, one writes:

$$\frac{\partial U}{\partial M_A} = 0\,. \tag{5}$$

After substituting Eq. (2) into Eq. (5), one obtains:

$$\frac{\partial U}{\partial M_A} = \int_0^{\pi/2} \frac{MR}{EI}\frac{\partial M}{\partial M_A} d\theta = 0\,, \tag{6}$$

where, from Eq. (1), it can be seen that:

$$\frac{\partial M}{\partial M_A} \equiv 1\,.$$

Thus, M_A can be obtained from:

$$\int_0^{\pi/2} M d\theta = 0. \qquad (7)$$

Substituting Eq. (1) into Eq. (7) and integrating it leads to:

$$M_A = \frac{FR}{2}(1 - \frac{2}{\pi}) \qquad (8)$$

Utilising Eq. (8) in Eq. (1), the moment M now becomes:

$$M = \frac{FR}{2}(\cos\theta - \frac{2}{\pi}). \qquad (9)$$

Combining Eq. (8), Eq. (4) and Eq. (3), then integrating them gives:

$$\delta = \frac{FR^3}{2EI}\left(\frac{\pi}{4} - \frac{2}{\pi}\right) \qquad (10)$$

This is the deflection of the upper right quadrant, which is the same as that of the upper left quadrant. Thus, the deflection of the upper half of the ring is given by Eq. (10). Doubling δ gives the total deflection of the ring, as follows:

$$y \equiv 2\delta = \frac{FR^3}{EI}\left(\frac{\pi}{4} - \frac{2}{\pi}\right). \qquad (11)$$

Since the ring is symmetrical about a horizontal axis through A, the lower half of the ring will deflect by the same amount as the upper half. Thus, the total deflection, y, is twice that given by Eq. (10) [1][2][6][8]. It can also be written as follows:

$$y = \frac{FD^3}{16EI}\left(\frac{\pi}{2} - \frac{4}{\pi}\right), \qquad (12)$$

where D = 2R denotes the diameter of the ring; F, the applied force; y, the change in the diameter of the ring in the direction of the force; E, Young's modulus of elasticity of the material of which the ring is made, and I, the moment of inertia of the cross-section of the ring about its centroidal axis of bending.

The result shown in Eq. (12) agrees with that tabulated by Beckwith and Marangoni [1]. This is a simplified version of that derived by Cook [8]. It follows from Eq. (12) that the ring behaves as a linear spring with a force, F, that is related to its deflection, y, by:

$$F = k_1 y, \qquad (13)$$

where:

$$k_1 \equiv \frac{16}{\left(\frac{\pi}{2} - \frac{4}{\pi}\right)} \frac{EI}{D^3} \qquad (14)$$

and

$$I \equiv \frac{\pi D^3 t}{4}. \qquad (15)$$

The local strain to be registered by the indicator is the change in the local arc length around the strain gauge divided by the original arc length. It can be written as follows:

$$\varepsilon = \frac{ds}{s}$$

where, from Figure 5, $s = R\theta$ and $ds = \frac{\partial s}{\partial R} dR + \frac{\partial s}{\partial \theta} d\theta$.

It follows from these two equations that:

$$\frac{ds}{s} = \frac{dR}{R} + \frac{d\theta}{\theta} \approx \frac{dR}{R}$$

Thus, the strain can be estimated using the ratio:

$$\varepsilon \approx \frac{y}{R}, \qquad (16)$$

where y is given by Eq. (12).

Using Eq. (16) in Eq. (13), then utilising Eq. (14) and Eq. (15), one obtains:

$$F = k\varepsilon \qquad (17)$$

where:

$$k \equiv \frac{2\pi DEt}{\left(\frac{\pi}{2} - \frac{4}{\pi}\right)} \qquad (18)$$

These last two equations suggest that, when designed and manufactured properly, a thin ring will behave as an elastic spring of stiffness k given by Eq. (18), that is one that exhibits a linear relationship between applied force and the resulting deflection, as in Eq. (13), or between the applied force and the resulting strain, as in Eq. (17).

The ring that was designed and built at Indiana University-Purdue University Fort Wayne is pictured in Figure 4. It is made of aluminium and had the following key dimensions:

- Outside diameter: $D_0 = 31.5mm$;
- Inside diameter: $D_i = 27.5mm$;
- Thickness: $t = 2.0mm$;
- Width of the ring: $w = 19.1mm$;
- Young's modulus of elasticity: $E = 6.89510^7 \, Pa$.

Introducing these dimensions into Eq. (15) and Eq. (18), the area moment of inertia, I, and the stiffness of the ring, k, are found to be, respectively:

- $I = 49.0966 \times 10^{-9} m^4$;
- $k = 0.09172 \times 10^6 N$.

The use of the numerical value of k in Eq. (17) turns this equation into:

$$F = 0.09172 \times 10^6 \varepsilon \times (10^{-6}). \qquad (19)$$

Since one expects the strain to be read in units of micro strains $\left(10^{-6} \times strain\right)$, one writes

$$F = 0.09172\varepsilon, \quad (20)$$

where the units of micro strains have already been accounted for. It follows that, when using the ring as a force transducer, the measured strain can be converted into the applied force that caused it by using Eq. (20).

TESTING THE RING AS A FORCE SENSOR

In order to evaluate Eq. (20), the ring was tested by hanging blocks of known masses on it. To this end, a set of cylindrical blocks was utilised with standardised masses purchased from Ohaus Corporation [11]. The blocks had hooks and allowed for variation in the masses that could be suspended onto the ring from zero to 2,000g, in 50-gram increments. For each test mass, the corresponding strain registered by the strain indicator was recorded. The weights of the test masses were plotted versus the corresponding strains, as shown in Figure 6. A straight line was fitted to the plotted data [14]. The resulting equation is:

$$F = 0.0912\varepsilon. \quad (21)$$

It is quite remarkable that the experimental result given by Eq. (21) is within 0.5% of that from the analysis in Eq. (20).

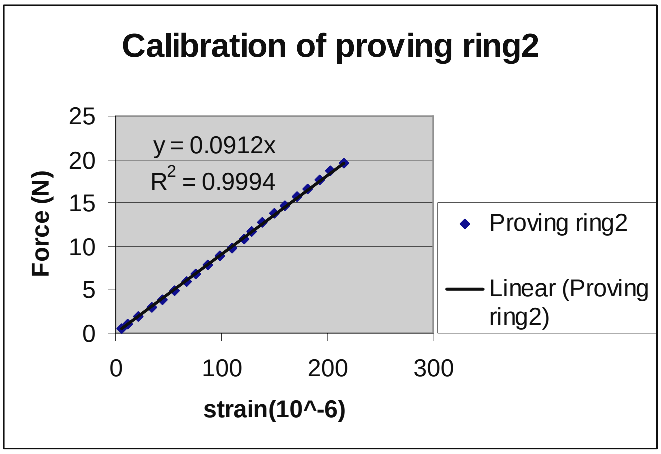

Figure 6: Elastic behaviour of the designed ring.

CONCLUSIONS

In this article, the author presents a project, the results of which have been integrated into instruction and research. A force sensor was needed for laboratory experiments. So several concepts were designed, analysed, built and tested as a class exercise. The most successful design was based upon the Whittmore-Petrenko proving ring. The elastic behaviour of the designed ring was analysed using Castigliano's second theorem and the results so obtained were found to be within 1% of those obtained from testing the ring.

The designed and calibrated ring is being used for instruction and research to determine the force exerted on a sphere by the surrounding fluid. When the fluid is at rest, the measured force is at buoyancy. But when there is relative motion between the immersed sphere and the surround fluid, the force represents viscous drag [13].

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